

# Green Functions of $\mathcal{N} = 1$ SYM and Radial/Energy-Scale Relation

Xiao-Jun Wang\*

*Institute of Theoretical Physics, Beijing, 100080, P.R. China*

Sen Hu†

*Department of Mathematics,  
University of Science and Technology of China,  
Anhui, Hefei, 230026, P.R. China*

We study counter-terms of one- and two-point Green functions of some special operators in  $\mathcal{N} = 1$  SYM from their SUGRA duals from the consideration of AdS/CFT or gauge/gravity correspondence. We consider both the Maldacena-Núñez solution and the Klebanov-Strassler-Tseytlin solution that are proposed to be SUGRA duals of  $\mathcal{N} = 1$  SYM. We obtain radial/energy-scale relation for each solution by comparing SUGRA calculations with the field theory results. Using these relations we evaluate the  $\beta$ -function of  $\mathcal{N} = 1$  SYM. We find that the leading order term can be accurately obtained for both solutions and the higher order terms exhibit some ambiguities. We discuss the origin of these ambiguities and conclude that more studies are needed to check whether these SUGRA solutions are exactly dual to  $\mathcal{N} = 1$  SYM.

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## I. INTRODUCTION

Due to the remarkable success of AdS/CFT correspondence [1] more and more attention has been attracted to the study of gauge/gravity duality with less supersymmetry and the gauge theory part is not conformally invariant. The  $d = 4$ ,  $\mathcal{N} = 1$  super Yang-Mills theory (SYM) turned out to be extremely interesting with rich results. Nonsingular supergravity (SUGRA) solutions are constructed and proposed to be dual of the  $\mathcal{N} = 1$  super Yang-Mills theory. With these solutions we are able to get information of the dual quantum gauge theory at all energy scales. So far two SUGRA backgrounds were found:

- The Maldacena-Núñez (MN) solution[2]: It corresponds to a large ( $N$ ) number of D5-branes wrapped on a supersymmetric two-cycle inside a Calabi-Yau threefold. It is dual to pure  $\mathcal{N} = 1$   $SU(N)$  SYM.
- The Klebanov-Strassler-Tseytlin (KST) solution [3, 4]: It describes the geometry of the warped deformed conifold when one places  $M$  D3-branes and  $N$  fractional D3-branes at the apex of the conifold. It is dual to a certain  $\mathcal{N} = 1$  supersymmetric  $SU(N + M) \times SU(M)$  gauge theory. If  $M$  is a multiple of  $N$ , then this theory flows to  $SU(N)$  in the IR, via a chain of duality cascade which reduces the rank of the gauge group by  $N$  units at each cascade jump. Thus at the end of the duality cascade the gauge theory is effectively pure  $\mathcal{N} = 1$   $SU(N)$  SYM.

The verification on the above gauge/gravity duality is to perform calculations for correlation functions in terms of the SUGRA duals, in particular for Green functions in the non-conformal case. Since the KST solution is asymptotically  $AdS_5 \times X^5$  in the UV, where  $X^5$  is an Einstein manifold, the extension to KST solution is direct[5]. That is the classical action of SUGRA is identified with the generating function of  $\mathcal{N} = 1$  SYM at the large  $N$  limit and bulk field/boundary operator correspondence is established via solving the SUGRA equations of motion of bulk field  $\phi(x, r)$  subject to boundary conditions in the UV, i.e., for  $r \rightarrow \infty$ ,  $\phi(x, r) = r^\Delta \phi_0(x)$  where  $r$  is radial of the asymptotic  $AdS$  space, and  $\Delta$  is related to (mass) dimension of the boundary operator. The extension to MN solution, however, has to be re-examined since MN solution is no longer asymptotic to  $AdS$ . MN background can be treated as a domain wall solution of a truncated  $d = 7$  gauge supergravity[6, 7] and this  $d = 7$  gauged supergravity is obtained by compactification of IIB supergravity on  $S^3$ . Thus the partition function of IIB superstring in the MN background can naturally be identified with the generating function of  $\mathcal{N} = 1$  SYM. At large  $N$  limit we can identify the classical SUGRA action in the MN background with the generating function of  $\mathcal{N} = 1$  SYM. The bulk field/boundary operator correspondence is also established via similar method as in KST solution. Because the transverse space of MN background is parameterized as  $S^2 \times S^3$  respectively in UV,  $\Delta$  is determined by two parameters which are eigenvalues of the spherical harmonic functions on  $S^2$  and on  $S^3$ . Meanwhile  $\Delta$  in KST solution is determined by the mass  $m$  of the bulk field in  $AdS_5$ .

When we try to extend gauge/gravity duality beyond the conformally invariant case with maximal supersymmetry a well-known problem is that fluctuation equations on those complicated background can not be strictly solved. It prevents us from extracting enough informa-

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\*Electronic address: wangxj@itp.ac.cn  
†Electronic address: shu@ustc.edu.cn

tion about the Green functions. In general there are two kinds of useful information included in Green functions from their SUGRA dual description. One is the singular behavior of the Green function which can be used to directly check the duality between SYM and their SUGRA dual. For KST solution it was partially discussed in ref.[5]. Another one is the structure of counter-terms of the Green function from which we can obtain information of the renormalization group. In Ref.[8] we have shown that it can be extracted even though we only got near-boundary asymptotic solution of the fluctuation equation. In other words with those asymptotic solution and the UV/IR relation[9] we can analyze the divergence structure of the Green function of SYM from their SUGRA dual. In particular the relation between the radial in SUGRA configuration and the energy scale in the dual gauge theory can in principle be established via comparing counter-terms of the Green functions calculated from SUGRA side and from SYM side respectively. In Ref.[8] the  $\mathcal{N} = 2$  case was successfully studied. The purpose of the present paper is to study the structure of counter-terms of the Green functions and the consequent radial/energy-scale relation with  $\mathcal{N} = 1$  supersymmetry in the framework of both MN and KST background.

The radial/energy-scale relation is crucial in the study of gauge/gravity duality. For conformally invariant case this relation has been well established[1, 9, 10]. For non-conformally invariant theories it was suspected that to establish such a relation may be difficult for the relation maybe ambiguous. However at least for  $\mathcal{N} = 2$  case it was shown that such a relation can be unambiguously obtained via several different ways[7, 8]. For  $\mathcal{N} = 1$  case although there are some complications many interesting results have been achieved: the authors of Ref.[11] extend radial/energy-scale relation to KST solution and show that it is right at the leading order. In Ref.[12], the leading order expression of radial/energy-scale relation in MN solution was also achieved via comparing  $\beta$ -functions calculated from both the SUGRA side and the field theory side. In particular the authors of Ref.[7] (DLM) suggested radial/energy-scale relation in  $\mathcal{N} = 1$  case can be completely established via considering the phenomena of gaugino condensation. In concrete they considered a scalar SUGRA field whose boundary value couples to a gaugino bi-linear form and they identified the scaling behavior of this field to the inverse of gaugino condensation. They have applied this idea to MN solution and obtain radial/energy scale relation for the MN solution and consequently obtain  $\beta$ -function for all loops. However in Ref.[13] it was shown that DLM's result is right at the leading order only and a correction has to be made to compensate the effect of gaugino condensation in order to get the right result for higher order. Surprisingly DLM's idea has been applied to KST solution[14] and the right  $\beta$ -function for all orders was obtained.

In this paper, following the method shown in Ref.[8], we calculate one- and two-point Green functions of cer-

tain operators of SYM via their SUGRA dual descriptions. These results in general involve a large radial cut-off. It corresponds to UV cut-off in quantum field theory (QFT) calculation. Therefore we can investigate the divergence behavior of Green functions resulted from SUGRA and from QFT respectively and compare coefficients of these terms possessing same pole behavior. Then we will obtain a radial/energy-scale relation for this gauge/gravity correspondence. Of course the  $\mathcal{N} = 1$  case is very different from the  $\mathcal{N} = 2$  case. We will discuss those differences in section II and in section IV.

The paper is organized as follows. In section II we consider the bilinear gaugino operator. We calculate gaugino condensation (one-point function) and two-point function of this operator for MN solution. We then extract radial/energy-scale relation for MN solution and calculate the  $\beta$ -function to check this relation. In section III the same steps will be performed for KST solution. However for KST solution we conveniently consider dimension four operator instead of the bilinear gaugino operator. In section IV we discuss some ambiguities when we try to exactly match results from SUGRA description with the one from QFT. We also devote a brief summary in this section.

## II. GREEN FUNCTIONS AND RADIAL/ENERGY-SCALE RELATION FOR MN SOLUTION

### A. The MN Solution

The MN solution can be obtained via finding a domain wall solution of a seven dimension gauge supergravity after truncating its  $SO(4)$  gauge group to  $SU(2)$ [7]. This solution is parameterized by three dimensionless scalar functions  $k$ ,  $g$ , and  $a$ . They are in general independent of the five transverse space dimensions whose geometry is  $S^3 \times S^2$ . Therefore we can further compactify this truncated 7-d supergravity on  $S^2$  and get a 5-d effective action

$$S = \eta \int d^4x d\rho \epsilon^{2k} \{ 4\partial_i k \partial^i k - 2\partial_i g \partial^i g - \frac{\partial_i a \partial^i a}{2} e^{-2g} + V \}, \quad (1)$$

where the metric is  $g_{ij} = \eta_{ij}$ ,  $\rho$  is a dimensionless radial parameter,  $\eta \sim \alpha'^{-2}$  and

$$V = 4 + 2e^{-2g} - \frac{(1-a^2)^2}{4} e^{-4g}. \quad (2)$$

The MN solution is obtained by assuming that functions  $k$ ,  $g$ , and  $a$  depend on  $\rho$  only. Explicitly we read off this solution in the ten-dimensional string frame metric [2, 7, 15]

$$\begin{aligned}
ds^2 &= e^\Phi \left[ dx_{1,3}^2 + \frac{e^{2g}}{\lambda^2} (d\theta^2 + \sin^2 \theta d\phi^2) + \frac{1}{\lambda^2} d\rho^2 + \frac{1}{4\lambda^2} \sum_{a=1}^3 \left( w^a - \frac{A^a}{\lambda} \right)^2 \right], \\
e^{2g} &= \rho \coth 2\rho - \frac{\rho^2}{\sinh^2 2\rho} - \frac{1}{4}, \\
e^{2\Phi} &= e^{2k-2g} = ce^{-g} \sinh 2\rho, \quad a = \frac{2\rho}{\sinh 2\rho},
\end{aligned} \tag{3}$$

where  $\lambda^{-2} = Ng_s \alpha'$ ,  $c$  are integral constants,  $w^a$  parameterizes the 3-sphere, and gauge field  $A_a$  are written as

$$A^1 = -\lambda a d\theta, \quad A^2 = \lambda a \sin \theta d\phi, \quad A^3 = -\lambda \cos \theta d\phi. \tag{4}$$

The value of the Yang-Mills coupling is given in terms of the volume of the sphere  $S^2$  by [7]

$$\frac{1}{g_{\text{YM}}^2} = \frac{\text{Vol}_{S^2}}{g_{D_6}^2} = \frac{N}{16\pi^2} Y(\rho) E \left( \sqrt{\frac{Y(\rho) - 1}{Y(\rho)}} \right), \tag{5}$$

where the function  $Y$  is defined as

$$Y(\rho) = 4e^{2g} + a^2 = 4\rho \coth 2\rho - 1, \tag{6}$$

and  $E$  is a complete elliptic integral

$$E(k) = \int_0^{\pi/2} dx \sqrt{1 - k^2 \sin^2 x}. \tag{7}$$

## B. Green functions and radial/energy scale relation

In the  $\mathcal{N} = 2$  case we studied Green functions of dimension four operator,  $\mathcal{O}_4(x) = \text{Tr}(F_{\mu\nu} F^{\mu\nu} + \dots)$ , which couples to fluctuation of gauge coupling. However it is difficult to consider similar operator for the MN solution. The reason is that the gauge coupling  $g_{\text{YM}}^{-2}$  is a complicated nonlinear function of fields  $g$  and  $a$ . Because of this it is very hard to identify fluctuation of gauge coupling to one of the dynamical fields  $k$ ,  $g$  or  $a$ . Consequently the fluctuation equation corresponding to the above operator can not be derived. Fortunately it is well-known that the field  $a$  directly couples to gaugino bilinear operator [11],

$$\mathcal{O}_3(x) = \text{Tr} \bar{\psi}(x) \psi(x). \tag{8}$$

Therefore it is convenient to calculate Green functions of  $\mathcal{O}_3(x)$ .

Let us consider the fluctuation of  $a$  in background (3),

$$a(x, \rho) \rightarrow \bar{a}(\rho) + \frac{\varphi(x, \rho)}{\lambda}, \tag{9}$$

where  $\bar{a}(\rho)$  is a background solution in Eq. (3),

$$\varphi(x, \rho) = s(\rho) \int \frac{d^4 p}{(2\pi)^4} e^{ip \cdot x} \tag{10}$$

is the fluctuation normalized to  $s(\rho \rightarrow \infty) = 1$  at the boundary. The field equation for this fluctuation reads off

$$\begin{aligned}
\ddot{s} + 2(\dot{k} - \dot{g})\dot{s} - (n + \frac{p^2}{\lambda^2})s + e^{-2g}(m - 3a^2)s \\
+ 2\bar{a}(\dot{\delta k} - \dot{\delta g}) - \bar{a}(1 - \bar{a}^2)e^{-2\delta g} = 0,
\end{aligned} \tag{11}$$

where dot denotes differential on  $\rho$ ,  $\delta k$  and  $\delta g$  are fluctuations of fields  $k$ , and  $g$  respectively and the constants  $m$  and  $n$  are associated to eigenvalues of the spherical harmonic functions of  $S^2$  and  $S^3$  respectively.

The one-point function of  $\mathcal{O}_3$  or the gaugino condensation depends on normalization of fluctuations yet it is independent of the explicit expression of fluctuation. Taking cut-off  $\rho_0 \rightarrow \infty$  at a region near the boundary we easily get

$$\langle \text{Tr} \bar{\psi} \psi \rangle = \frac{\eta}{\lambda} e^{2k-2g} \dot{a} \Big|_{\rho=\rho_0} = \frac{c\eta}{\lambda} \rho_0^{1/2} + O(\rho_0^{-1/2}) \tag{12}$$

The gaugino condensation of  $\mathcal{N} = 1$  SYM has been computed in terms of quantum field theory methods in different schemes [16, 17, 18]. In the Pauli-Villars scheme it is

$$\langle \text{Tr} \bar{\psi} \psi \rangle_{\text{QFT}} = \text{Const.} \frac{1}{g_{\text{YM}}^2} \Lambda^3 e^{-\frac{8\pi^2}{N g_{\text{YM}}^2}}, \tag{13}$$

where  $\Lambda$  is an UV cut-off of QFT. In addition for large  $\rho$  the gauge coupling is approximately

$$\frac{4\pi^2}{N g_{\text{YM}}^2} = \rho + \frac{1}{16} \ln \rho + c + O\left(\frac{\ln \rho}{\rho}\right). \tag{14}$$

It implies that the large  $\rho$  limit corresponds to weak 't Hooft coupling limit of the gauge theory and it agrees with asymptotic freedom of the  $\mathcal{N} = 1$  pure SYM. Using Eq. (14), the gaugino condensation (12) can be rewritten as

$$\begin{aligned}
\langle \text{Tr} \bar{\psi} \psi \rangle_{\text{SUGRA}} &= \frac{4\pi^2 c \eta}{\lambda N} \rho_0^{-3/8} e^{2\rho_0} \frac{1}{g_{\text{YM}}^2} e^{-\frac{8\pi^2}{N g_{\text{YM}}^2}} \\
&= Z l_0^{-3} \frac{1}{g_{\text{YM}}^2} e^{-\frac{8\pi^2}{N g_{\text{YM}}^2}},
\end{aligned} \tag{15}$$

where we define a new radial variable  $l^{-3} = c\lambda^3 \rho^{-3/8} e^{2\rho}$  and take cut-off  $l = l_0$ . The constants  $Z$  and  $l$  are independent of  $\alpha'$ . Comparing Eqs. (13) with (15) we have

$$l_0^{-3} \sim \Lambda^3. \tag{16}$$

Now let both the radial and the momentum flow away from their cut-off point we obtain radial/energy-scale relation for the MN solution

$$\rho^{-3/8} e^{2\rho} \sim \frac{\mu^3}{M^3}, \quad (17)$$

where  $M$  is a definite energy scale.

In order to calculate the two-point function of  $\mathcal{O}_3$  we have to solve Eq. (11) explicitly. However equation (11) does not have an explicit analytic solution. Actually for our purpose we do not need the full analytic solution we only need asymptotic solution of fluctuation close to the boundary (or large  $\rho$ ). Notice that both  $\delta k$  and  $e^{\delta g}$  are normalized at the boundary, i.e.,  $\delta k \sim e^{\delta g} \sim 1$  for  $\rho \rightarrow \rho_0$  and  $\bar{a} \sim e^{-2\rho}$ . Then up to  $O(e^{-2\rho})$ ,  $s$  is decoupled from  $\delta k$  and  $\delta g$ . Therefore for  $\rho \rightarrow \infty$  the field equation (11) is approximately

$$\ddot{s} + (2 - \frac{1}{2\rho})\dot{s} - (n + \frac{p^2}{\lambda^2} - \frac{m}{\rho})s = 0. \quad (18)$$

The asymptotic solution of fluctuation field is

$$\begin{aligned} \varphi(x, \rho) &\xrightarrow{\rho \rightarrow \infty} \frac{G(\rho)}{G(\rho_0)} e^{ip \cdot x} \\ G(\rho) &= \sum_{i=1}^2 \lambda_i \rho^{\frac{a_i - 2m}{4(a_i + 1)}} e^{a_i \rho}, \end{aligned} \quad (19)$$

with constants  $\lambda_i$  and

$$\begin{aligned} a_1 &= -1 + \sqrt{1 + n + \frac{p^2}{\lambda^2}}, \\ a_2 &= -1 - \sqrt{1 + n + \frac{p^2}{\lambda^2}}. \end{aligned} \quad (20)$$

In general  $\lambda_i$  are not constants but are functions of  $p^2/\lambda^2$  [5]. As we will see in eq. (24), however,  $\lambda_i$  do not contribute to two-point Green function when we take cut-off to infinity. Because of this we will conveniently treat  $\lambda_i$  as constants.

Because the operator  $\mathcal{O}_3$  is protected at quantum correction we expect fluctuation field  $\varphi$  to be (mass) dimension one. In terms of radial/energy-scale relation (17) it implies

$$a_1 = \frac{2}{3} + O(\frac{p^2}{\lambda^2}), \quad \frac{a_1 - 2m}{4(a_1 + 1)} = -\frac{1}{8} + O(\frac{p^2}{\lambda^2}), \quad (21)$$

with

$$m = \frac{1}{2}, \quad n = \frac{16}{9}, \quad (22)$$

and  $a_2 = -\frac{8}{3} + O(p^2/\lambda^2)$ . The solution (19) is simplified to

$$\begin{aligned} G(\rho) &= \lambda_1 \rho^{-\frac{1}{8}} e^{2\rho/3} \left( 1 + \frac{p^2}{\lambda^2} \left( \frac{3}{10} \rho + \frac{27}{500} \ln \rho \right) \right. \\ &\quad \left. + O(e^{-10\rho/3}) + O(p^4/\lambda^4) \right). \end{aligned} \quad (23)$$

Now let us check contribution from fluctuation fields  $\delta k$  and  $\delta g$ . As discussed previously we should add  $O(e^{-2\rho})$  terms in  $G(\rho)$  if we consider their contribution. These terms are  $O(e^{-8\rho/3})$  comparing with the leading order of  $G(\rho)$  and therefor we can consistently ignore those terms.

Using solution (19) and (23) the two-point function is

$$\begin{aligned} \langle \mathcal{O}_3(p) \mathcal{O}_3(q) \rangle &= \text{const.} + \delta^4(p+q) \frac{3c\eta}{20\lambda^2} \left( \frac{p^2}{\lambda^2} + O(p^4/\lambda^4) \right) \left( 1 + \frac{9}{50\rho_0} \right) \rho_0^{-1/2} e^{2\rho_0} + O(e^{-2\rho_0/3}) \\ &= \text{const.} + \delta^4(p+q) p^2 \frac{3}{20} Z \frac{\Lambda^3}{\lambda^3} [1 + \frac{9}{50} (\ln \frac{\Lambda^2}{M^2})^{-1}] + O(p^4/\lambda^4) + O(1/\Lambda). \end{aligned} \quad (24)$$

From this result we see that there are no finite terms in two-point function of  $\mathcal{O}_3$ . At low energy  $O(p^4)$  terms are suppressed by  $\alpha'$ . Then only  $p^2$  terms survive. The above result is important: it implies that gauginos is a dynamical composite field, like quarks composed by mesons in QCD.

### C. Further discussions

Using Eqs. (14) and (17) we obtain  $\beta$ -function of pure  $\mathcal{N} = 1$  SYM

$$\beta(g_{\text{YM}}) = -\frac{3N}{16\pi^2} g_{\text{YM}}^3 [1 + \frac{N g_{\text{YM}}^2}{16\pi^2} + O(g_{\text{YM}}^4)]. \quad (25)$$

On the other hand the  $\beta$ -functions obtained by field theory method (NSVZ) [19] is

$$\beta(g_{\text{YM}})_{\text{NSVZ}} = -\frac{3N}{16\pi^2} g_{\text{YM}}^3 [1 - \frac{N g_{\text{YM}}^2}{8\pi^2}]^{-1}. \quad (26)$$

The leading order term of the expression (25) ( $g_{\text{YM}}^3$ ) precisely agree with result of field theory. The sub-leading order terms, however, does not match with the result of NSVZ. This is a puzzle of our results. We will see that similar results also appear for the KST solution. In section IV we will discuss several possible solutions for this puzzle.

Another important issue shown in this section is how to establish field/operator correspondence for the MN solution. For the sake of convenience let us consider scalar field as an example: All scalar fields in 5-d bulk are distinguished by two eigenvalues of the spherical harmonic functions of the transverse space  $S^2 \times S^3$  in the UV. It is different from AdS/CFT correspondence that only one eigenvalue generate “mass” of bulk field here we need both eigenvalues to determine the scaling behavior of bulk field in the UV. This scaling behavior associates with (mass) dimension of the dual operator of the boundary quantum field theory. Therefore field/operator correspondence can be unambiguously established via correspondence between (mass) dimension of operator and eigenvalues of the spherical harmonic functions of the transverse space  $S^2 \times S^3$ .

### III. GREEN FUNCTIONS AND RADIAL/ENERGY-SCALE RELATION FOR THE KST SOLUTION

#### A. The KST solution

The KST solution is proposed to be dual of  $\mathcal{N} = 1$  pure SYM with gauge group  $SU(N + M) \times SU(M)$ . It is realized via placing  $M$  D3-branes and  $N$  fractional D3-branes on the conifold [21]. If  $M$  is a multiple of  $N$  then this theory flows to  $SU(N)$  in the IR via a chain of duality cascade which reduces the rank of the gauge group by  $N$  units at each cascade jump. Thus at the end of the duality cascade the gauge theory is effectively the  $\mathcal{N} = 1$   $SU(N)$  pure SYM. It was shown in [4] that in order to remove the naked singularity found in [3] the conifold have to be replaced by the deformed one. The 10-d metric in the string frame takes “D-brane” form:

$$ds_{10}^2 = h^{-1/2}(\tau) dx_n dx_n + h^{1/2}(\tau) ds_6^2, \quad (27)$$

where  $\tau$  is the radial parameter of the transverse space.  $ds_6^2$  is the metric of the deformed conifold[22],

$$ds_6^2 = \frac{1}{2} \epsilon^{4/3} K(\tau) \left[ \frac{1}{3K^3(\tau)} (d\tau^2 + (g^5)^2) \right]$$

$$+ \cosh^2\left(\frac{\tau}{2}\right) \sum_{i=3}^4 (g^i)^2 + \sinh^2\left(\frac{\tau}{2}\right) \sum_{i=1}^2 (g^i)^2 \right] \quad (28)$$

where  $\epsilon$  is a parameter with length dimension  $3/2$ ,  $K(\tau)$  is given by

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3} \sinh \tau}, \quad (29)$$

and the 1-form basis  $g^i$  is defined as follows

$$\begin{aligned} g^1 &= -\frac{1}{\sqrt{2}}(\sin \theta d\phi_1 + \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2), \\ g^2 &= \frac{1}{\sqrt{2}}(d\theta_1 - \sin \psi \sin \theta_2 d\phi_2 - \cos \psi d\theta_2), \\ g^3 &= -\frac{1}{\sqrt{2}}(\sin \theta d\phi_1 - \cos \psi \sin \theta_2 d\phi_2 + \sin \psi d\theta_2), \\ g^4 &= \frac{1}{\sqrt{2}}(d\theta_1 + \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2), \\ g^5 &= d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2. \end{aligned} \quad (30)$$

For the KST solution the type IIB SUGRA fields NS-NS 2-form  $B - 2$ , R-R 2-form  $C_2$  and the dilaton  $\Phi$  are excited. The simplest ansatz for the 2-form fields is

$$B_2 = \frac{g_s N \alpha'}{2} [f(\tau) g^1 \wedge g^2 + \kappa(\tau) g^3 \wedge g^4], \quad (31)$$

$$F_3 = \frac{N \alpha'}{2} \{g^5 \wedge g^3 \wedge g^4 + d[F(\tau)(g^1 \wedge g^3 + g^2 \wedge g^4)]\},$$

with boundary condition  $F(0) = 0$  and  $F(\infty) = 1/2$ . The self-dual 5-form field strength may be decomposed as  $\hat{F}_5 = \mathcal{F}_5 + * \mathcal{F}$ ,

$$\begin{aligned} \mathcal{F}_5 &= B_2 \wedge F_3 = \frac{g_s N \alpha'^2}{4} l(\tau) g^1 \wedge g^2 \wedge g^3 \wedge g^4 \wedge g^5, \\ l(\tau) &= f(1 - F) + \kappa F. \end{aligned} \quad (32)$$

Writing the dilation field as

$$e^\Phi = e^{\Phi_0 + \varphi} = g_s e^\varphi, \quad (33)$$

and define

$$\alpha = 4(g_s N \alpha')^2 \epsilon^{-8/3}, \quad (34)$$

the type IIB SUGRA field equations can be rewritten as follows

$$\begin{aligned} 2h \frac{d}{d\tau} (\epsilon^\varphi h^{-1} F') + \epsilon^\varphi (1 - F) \tanh^2\left(\frac{\tau}{2}\right) - \epsilon^\varphi F \coth^2\left(\frac{\tau}{2}\right) &= \alpha (\kappa - f) \frac{l}{K^2 h \sinh^2 \tau}, \\ h \frac{d}{d\tau} (\epsilon^\varphi h^{-1} \coth^2\left(\frac{\tau}{2}\right) f') - \frac{1}{2} \epsilon^\varphi (f - \kappa) &= \alpha \frac{l(1 - F)}{K^2 h \sinh^2 \tau}, \end{aligned}$$

$$\begin{aligned}
h \frac{d}{d\tau} (e^\varphi h^{-1} \tanh^2(\frac{\tau}{2}) \kappa') + \frac{1}{2} e^\varphi (f - \kappa) &= \alpha \frac{lF}{K^2 h \sinh^2 \tau}, \\
\nabla^2 \varphi &= \frac{\alpha}{8} e^{-4/3} \left\{ \frac{(1-F)^2 e^\varphi - \kappa'^2 e^{-\varphi}}{\cosh^4(\frac{\tau}{2})} + \frac{F^2 e^\varphi - f'^2 e^{-\varphi}}{\sinh^4(\frac{\tau}{2})} + 8 \frac{F'^2 e^\varphi - (f - \kappa)^2 e^{-\varphi}}{\sinh^2 \tau} \right\} \\
\frac{1}{\sinh^2 \tau} \frac{d}{d\tau} (h' K^2 \sinh^2 \tau) &= -\frac{\alpha}{8} \left\{ \frac{(1-F)^2 e^\varphi - \kappa'^2 e^{-\varphi}}{\cosh^4(\frac{\tau}{2})} + \frac{F^2 e^\varphi - f'^2 e^{-\varphi}}{\sinh^4(\frac{\tau}{2})} + 8 \frac{F'^2 e^\varphi - (f - \kappa)^2 e^{-\varphi}}{\sinh^2 \tau} \right\}.
\end{aligned} \tag{35}$$

Setting  $\varphi = 0$  we get the KST solution:

$$\begin{aligned}
F(\tau) &= \frac{\sinh \tau - \tau}{2 \sinh \tau}, \\
f(\tau) &= \frac{\tau \coth \tau - 1}{2 \sinh \tau} (\cosh \tau - 1), \\
\kappa(\tau) &= \frac{\tau \coth \tau - 1}{2 \sinh \tau} (\cosh \tau + 1), \\
h(\tau) &= \frac{2^{2/3}}{4} \alpha \int_\tau^\infty dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3}.
\end{aligned} \tag{36}$$

It should be pointed out that  $f$  and  $\kappa$  can be shifted by the same constant. It yields the singular behavior of the metric for small  $\tau$  [4].

## B. Green functions

Since NS-NS 2-form is turned on in the KST solution the gauge coupling of dual SYM is

$$\frac{1}{g_{\text{YM}}^2} = \frac{1}{4\pi^2 \alpha' g_{D3}^2} \int d^2 \xi \sqrt{\det(G_{ij} + B_{ij})}. \tag{37}$$

In the present case for D5-branes wrapped on the vanishing  $S^2$  one has  $G = 0$ . From Eq. (30) and (31) we obtain

$$\frac{1}{g_{\text{YM}}^2} = \frac{N}{4\pi^2} \kappa(\tau). \tag{38}$$

Therefore we will consider Green functions of the following operator

$$\mathcal{O}_4(x) = -\frac{1}{4} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) + \text{Tr}(\bar{\psi} \mathcal{D} \psi). \tag{39}$$

In terms of AdS/CFT correspondence it requires us to solve fluctuation equation of  $\kappa$ . From Eq. (36) however we can see that functions  $\kappa$ ,  $f$ ,  $F$  and  $\varphi$  are not independent from each other. Then we have to solve simultaneous equations of all fluctuations  $\delta\kappa$ ,  $\delta f$ ,  $\delta F$  and  $\delta\varphi$ . The fluctuation equations do not admit full analytic solutions. We again consider near-boundary (or large  $\tau$ ) solution only. In the following all of our discussions and calculations should be treated at large  $\tau$  limit if we do not stay otherwise.

Conveniently we assume that the fluctuation  $\delta F$  vanishes (in the following we can see this assumption is a consistent one). Because  $e^{\delta\varphi}$  should be normalized at

the boundary, i.e.,  $e^{\delta\varphi} \rightarrow 1$  for  $\tau \rightarrow \infty$ , the fluctuation equations can be expanded to the first linear order for large  $\tau$ . From the fourth equation of Eq. (35) we get

$$\nabla^2 \delta\varphi \sim \alpha e^{-4/3} e^{-2\tau} [\delta\varphi - (\delta\kappa)' - (\delta f)'] + O(e^{-4\tau}). \tag{40}$$

Then noticing  $\delta\kappa$  and  $\delta f$  are also normalized at the boundary we have

$$\delta\varphi \xrightarrow{\tau \rightarrow \infty} P(\tau) e^{-2\tau} + O(e^{-4\tau}), \tag{41}$$

where  $P(\tau)$  is a polynomial function of  $\tau$ . Using Eq. (41) we find that the left side of fluctuation equation yielded from the first equation of (35) is of order  $e^{-2\tau}$ . Meanwhile the right side of this equation is proportional to

$$\frac{1}{\tau} (f\delta f - \kappa\delta\kappa) + \frac{1}{\tau^2} (f^2 - \kappa^2) \delta h. \tag{42}$$

Because  $f - \kappa \sim e^{-\tau}$  up to  $O(e^{-\tau})$  we may set  $\delta f = \delta\kappa = y(x, \tau)$ . Then fluctuation equation yielded from the last equation of (35) reduces to

$$\frac{d}{d\tau} [(\delta h)' e^{4\tau/3}] \sim \alpha y'. \tag{43}$$

Again, recalling  $y' \sim O(1/\tau)$ , we have  $\delta h \sim O(e^{-4\tau/3}/\tau)$ . Finally, up to  $O(e^{-\tau})$ , we obtain the fluctuation equation on  $y$  ( $\delta\kappa$ ) as follows

$$h \frac{d}{d\tau} (h^{-1} y') + \tilde{\alpha} \tau e^{-2\tau/3} \partial_m \partial^m y = (m^2 + \frac{4}{3\tau}) y, \tag{44}$$

where  $\tilde{\alpha} = 2^{-10/3} \epsilon^{4/3} \alpha$  and mass  $m$  is related to eigenvalues of the Laplace equation on  $X^5 = T^{1,1}$ .

We are considering a (mass) dimension four operator. As in AdS/CFT correspondence the scaling behavior of  $y(x, \tau)$  in the UV requires  $m = 0$ . Then asymptotic solution of Eq. (44) is

$$\begin{aligned}
y(x, \tau) &\xrightarrow{\tau \rightarrow \infty} \frac{G(\tau)}{G(\tau_0)} e^{ip \cdot x}, \\
G(\tau) &= [\tau - \frac{3}{4} + O(1/\tau)] + \frac{3}{8} \tilde{\alpha} p^2 [\tau^3 + O(\tau^2)] e^{-2\tau/3} \\
&\quad + O(p^4 e^{-4\tau/3}),
\end{aligned} \tag{45}$$

where  $\tau_0 \rightarrow \infty$  is a cut-off.

The 5-d action can be obtained via compactifying type IIB SUGRA on  $(g^1, \dots, g^5)$ . In particular only terms

involving derivatives of  $\tau$  contribute to one- and two-point functions:

$$S_5 = \eta \int d^4x d\tau \frac{1}{\tau} e^{4\tau/3} \kappa'^2 + \dots, \quad (46)$$

where  $\eta \sim N^2/\alpha'^2$ . Then the one-point function of  $\mathcal{O}_4$  is

$$\langle \mathcal{O}_4 \rangle = 2\eta \frac{1}{\tau_0} e^{4\tau_0/3}, \quad (47)$$

and the two-point function is

$$\begin{aligned} & \langle \mathcal{O}_4(p) \mathcal{O}_4(q) \rangle \\ &= \text{const.} + \frac{3}{4} \eta \tilde{\alpha} [1 + O(1/\tau_0)] e^{2\tau_0/3} p^2 \delta^4(p+q) \\ & \quad + \lambda_1 \eta \tilde{\alpha}^2 \tau_0^2 p^4 \delta^4(p+q) + O(\alpha' p^6), \end{aligned} \quad (48)$$

where  $\lambda_1$  is a constant.

### C. Radial/energy relation and anomalous dimension

For large  $\tau$  we may introduce another radial coordinate  $r$  via  $r = \epsilon^{2/3} e^{-\tau/3} [4]$ . Noticing  $\tilde{\alpha} \propto (g_s N \alpha')^2 \epsilon^{-4/3}$  and

$$\frac{1}{g_{\text{YM}}^2} = \frac{N}{8\pi^2} (\tau - 2), \quad \tau \rightarrow \infty, \quad (49)$$

the two-point function in Eq. (48) can be rewritten as

$$\begin{aligned} & \langle \mathcal{O}_4(p) \mathcal{O}_4(q) \rangle \\ & \sim (g_{\text{YM}}^2 N)^2 \delta^4(p+q) [Z_1 \ln r_0 (\ln r_0 + C) r_0^{-2} p^2 \\ & \quad + Z_2 \alpha' \epsilon^{-8/3} (\ln r_0)^4 p^4], \end{aligned} \quad (50)$$

where  $r_0 \rightarrow 0$  is a cut-off,  $Z_1$ ,  $Z_2$  and  $C$  are constants independent of  $\alpha'$ . Meanwhile the two-point function is easy to compute at the leading order by perturbative method of quantum field theory,

$$\begin{aligned} \langle \mathcal{O}_4(p) \mathcal{O}_4(q) \rangle & \sim (g_{\text{YM}}^2 N)^2 [a_0 \Lambda^4 + a_1 p^2 \Lambda^2 \\ & \quad + a_2 p^4 \ln \Lambda^2 + \dots] \delta^4(p+q). \end{aligned} \quad (51)$$

Comparing  $p^2$  terms in Eqs. (48) and in (51) we obtain the radial/energy-scale relation

$$\ln \mu/M = \frac{\tau}{3} + \ln \tau + \text{constant}. \quad (52)$$

From Eqs. (52) and (49) we have the following  $\beta$ -function

$$\beta(g_{\text{YM}}) = -\frac{3N}{16\pi^2} g_{\text{YM}}^3 [1 - \frac{3N}{8\pi^2} g_{\text{YM}}^2 + O(g_{\text{YM}}^4)]. \quad (53)$$

Similar as for MN solution we only recover the leading order term of NSVZ  $\beta$ -function. The sub-leading order terms, unfortunately, is very different from that of NSVZ  $\beta$ -function. In the next section we will show that there

are some ambiguities which makes it impossible to exactly determine the radial/energy scale relation beyond leading order in terms of the above method.

Another puzzle is related to the  $p^4$  term of two-point functions. First we see that the  $p^4$  term yielded from SUGRA description (Eq. (50)) is suppressed by  $\alpha'$  but not suppressed from field theoretical method (Eq. (51)). In fact it indicates that  $\epsilon^{8/3} \sim \alpha'$ . For KST solution the singularity of the conifold is removed through the blowing-up of the  $S^3$  of  $T^{1,1}$ . The length of  $S^3$  is parameterized by  $\epsilon$ . Therefore  $\epsilon^{8/3} \sim \alpha'$  means that the 3-sphere is very small. Recalling  $r_0 \sim \Lambda/M$  the  $p^4$  term yielded by SUGRA description is proportional to  $(\ln \Lambda^2/M^2)^4$ . However, the one obtained from perturbative calculation of QFT is proportional to  $\ln \Lambda^2/M^2$ . In the next section, we will show that, the Green functions yielded from SUGRA description represent non-perturbative results of QFT. It must correct the perturbative results of QFT.

In addition, there is a very interesting result hiding in the solution (45): The dimensions of  $\mathcal{O}_4$  requires that the leading order function  $G(\tau)$  should be a constant. However it is not true. From eqs. (45), (49) and (52) we see that the scaling behavior of  $G(\tau)$  is

$$G \sim 1 + \frac{3}{\tau_0} \ln \mu \simeq \mu^{\frac{3g_{\text{YM}}^2 N}{8\pi^2}}. \quad (54)$$

It indicates that  $\mathcal{O}_4$  has anomalous dimension

$$\gamma = \frac{3g_{\text{YM}}^2 N}{8\pi^2}. \quad (55)$$

In principle this method can be generalized to calculate anomalous dimensions of any operators.

## IV. DISCUSSION

In the previous two sections we have obtained counter-terms of some Green functions and  $\beta$ -function of  $\mathcal{N} = 1$  pure SYM from their SUGRA dual description. However there are some mismatches between the results from SUGRA description and those from perturbative calculations of QFT. In order to explain these mismatches we should first ask:

- Are MN solution or KST solution exactly dual to  $\mathcal{N} = 1$  SYM at large N limit?
- Is the classical action of SUGRA exactly equal to the generation function of SYM at large N limit?

In the case of SUGRA part with maximum supersymmetry and the gauge theory part conformally invariant the answer should be unambiguous. In the case of SUGRA part with less supersymmetry and the gauge theory part not conformally invariant, however, it is still very difficult to answer these questions thus far. It requires more careful studies on this type gauge/gravity duality. In

the conformally invariant case we knew that the property of weak/strong duality prevents us to directly check AdS/CFT correspondence. The same difficulty also appears our investigation. It induces ambiguity when we study the divergence structure of Green functions.

In order to illustrate this ambiguity, let us assume that the gauge/gravity duality is exact at large  $N$  limit for MN solution or KST solution. It is unambiguous that the SUGRA description reproduces non-perturbative results of QFT. It indicates that the Green functions obtained

from SUGRA description should contain all order terms of perturbative calculation of QFT. Usually the high orders can be ignored at weak 't Hooft coupling limit. It is not true for the divergence part because higher orders of 't Hooft coupling are more divergent than lower order. Let us consider KST solution as an example. Recalling  $\tau_0 \sim \ln \Lambda^2/M^2$  Eq. (49) indicates  $(g_{\text{YM}}^2 N) \ln \Lambda^2/M^2$  fixed. The perturbative calculation of QFT tells us that the most divergence part of two-point function of  $\mathcal{O}_4$  has the following form

$$\langle \mathcal{O}_4(p) \mathcal{O}_4(q) \rangle \sim \sum_{n=0}^{\infty} (g_{\text{YM}}^2 N)^{(n+2)} \left( \ln \frac{\Lambda^2}{p^2} \right)^n (a_n p^2 \Lambda^2 + b_n p^4 \ln \frac{\Lambda^2}{p^2} + \cdots) \delta^4(p+q). \quad (56)$$

This implies contributions of every order are equally important! Of course it is impossible to sum over all order contributions in terms of perturbative method of QFT. This is a potential origin of mismatches mentioned above. However we see that the structure of  $p^2 \Lambda^2$  in two-point Green function of  $\mathcal{O}_4$  is universal. We then obtain the right leading order result on  $\beta$ -function. It implies that the SUGRA solution should include right one-loop effect of dual SYM at least but the mismatch between Eq. (50) and (51) implies the SUGRA solution must include many higher loop effects of dual SYM.

In order to further understand the above ambiguity, we should notice that the divergence terms in Green functions is unphysical, i.e., it has to be subtracted via renormalization procedure. But this procedure introduces an extra freedom of energy scale described by the renormalization group equation. We can in general assume that  $p^2$  terms in two-point functions  $G^{(2)}(p)$  of any operator  $\mathcal{O}$  have the following form,

$$\begin{aligned} G^{(2)}(p^2, \Lambda)_{\text{QFT}} &\sim \sum_{n=0}^{\infty} (g_{\text{YM}}^2 N)^{(n+m)} \left( \ln \frac{\Lambda^2}{m^2} \right)^n a_n p^2 \Lambda^2, \\ G^{(2)}(p^2, \tau_0)_{\text{SUGRA}} &\sim Z_1 P(\tau_0) e^{a\tau_0} p^2, \end{aligned} \quad (57)$$

where  $\tau$  is related to the radial parameter of SUGRA configuration,  $\tau_0 \rightarrow \infty$  is a cut-off, and  $P(\tau_0) = \tau_0^l + \cdots$  is a polynomial function of  $\tau_0$ .

Now let both cut-off  $\Lambda$  and  $\tau_0$  have small changes, i.e.,  $\Lambda \rightarrow b\Lambda$ ,  $\tau_0 \rightarrow \tau_0 + c$  with  $|b-1| \ll 1$  and  $|c| \ll 1$ . It should be careful that  $g_{\text{YM}}$  in  $G^{(2)}(p)_{\text{QFT}}$  is also affected by a rescaling of  $\Lambda$ . This effect can be simply captured via treating  $g_{\text{YM}}$  as energy scale dependent, i.e.,  $g_{\text{YM}} = g_{\text{YM}}(\nu_0)$ , and  $\nu_0 \rightarrow b\nu_0$ . When cut-offs are removed we expect to exactly match the results obtained from SUGRA and QFT respectively. In other words we should have

$$\frac{G^{(2)}(p^2, b\Lambda)_{\text{QFT}}}{G^{(2)}(p^2, \Lambda)_{\text{QFT}}} = \frac{G^{(2)}(p^2, \tau_0 + c)_{\text{SUGRA}}}{G^{(2)}(p^2, \tau_0)_{\text{SUGRA}}}$$

$$\begin{aligned} &\Rightarrow \frac{\sum_{n=0}^{\infty} [g_{\text{YM}}^2(b\nu_0)N]^{(n+m)} \left( \ln \frac{\Lambda^2}{m^2} + \ln b^2 \right)^n a_n b^2}{\sum_{n'=0}^{\infty} [g_{\text{YM}}^2(\nu_0)N]^{(n'+m)} \left( \ln \frac{\Lambda^2}{m^2} \right)^n a_{n'}} b^2 \\ &= \frac{P(\tau_0 + c)}{P(\tau_0)} e^{ac}. \end{aligned} \quad (58)$$

Using result of renormalization group, we can write

$$g_{\text{YM}}^2(b\nu_0) = g_{\text{YM}}^2(\nu_0) (1 + \beta_0 g_{\text{YM}}^2(\nu_0) \ln b^2 + O(g_{\text{YM}}^4(\nu_0))) \quad (59)$$

where  $\beta_0$  is defined by  $\beta$ -function

$$\beta = \beta_0 g_{\text{YM}}^3 + \cdots$$

Then the second line of Eq. (58) is rewritten as

$$w = \frac{b^2 (1 + w g^2(\nu_0) \ln b^2), \sum_n a_n (N/s_0)^n [(n+m)\beta_0 + ns_0]}{\sum_{n'} a_{n'} (N/s_0)^{n'}}. \quad (60)$$

Here we have used the relation  $s_0 g^2(\nu_0) \ln \Lambda^2/m^2 \simeq 1$  and  $s_0$  depends on SUGRA solution. For MN solution,  $\rho_0 \simeq \ln \Lambda/m$ , then Eq. (19) gives  $s_0 = \frac{N}{8\pi^2}$ . For KST solution,  $\tau_0 \simeq 3 \ln \Lambda/m$ , then Eq. (49) tells us  $s_0 = \frac{3N}{16\pi^2}$ . Let  $b = \mu/m$  we can produce a freedom of energy scale, and  $c$  should be related to radial parameter of SUGRA solutions (named " $\tau$ "). Then we obtain the radial/energy-scale relation as follows

$$c \simeq \frac{1}{a} (1 + w g^2(\nu_0) - \frac{l}{a\tau_0}) \ln \frac{\mu^2}{m^2}. \quad (61)$$

For MN solution, unfortunately,  $w$  is not computable so that we can not obtain radial/energy-scale relation beyond the leading order. It just reflects the ambiguity mentioned above.

The ambiguity, however, still exists even though Eq. (60) is computable (e.g., for KST solution). The difficulty is how to exactly determine the relation between  $c$  and  $\tau$ . There are two natural conditions:  $c(\tau =$



$\tau_0) = 0$  and  $dc/d\tau(\tau \rightarrow \infty) = 1$ , but they are not sufficient to fix the relation between  $c$  and  $\tau$  beyond leading order for finite cut-off. For example, the relation  $c = (1 + x/\tau_0)(\tau - \tau_0)$  is allowed, but  $x$  is an unknown constant.

In conclusion: We have discussed field/operator correspondence in gauge/gravity duality in the MN and the KST background. In terms of this correspondence the counter-terms of some one- and two-point Green functions of  $\mathcal{N} = 1$  SYM have been studied via their SUGRA dual solution. Although we can only obtain asymptotical solution of fluctuation equations it is sufficient to determine some information of the renormalization group equation of  $\mathcal{N} = 1$  SYM. In particular the leading order behavior of  $\beta$ -function and anomalous dimension of  $\mathcal{O}_4$

are obtained. However some ambiguities appear when we want to obtain information beyond the leading order. It was shown that the extra constraints are needed to fix these ambiguities. It indicates that we need to obtain full analytical solutions instead of an asymptotical solutions of the fluctuation equations. This suggests more studies are needed to check whether the MN solution or the KST solution are exactly dual to  $\mathcal{N} = 1$  SYM.

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